

# ***Admittance model for (superconducting) magnets for Power Converters control***

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**September 26<sup>th</sup> 2018**

**POCPA 2018 - Session 5**



# Concept of Inductance: amplitude domain

- Several definitions exist for the inductance (of a magnet) [1]

- with respect to the current

$$L_a = \frac{\phi}{i} : \text{apparent or secant inductance}$$

$$L_d = \frac{d\phi}{di} = \frac{v - Ri}{di/dt} : \text{differential or incremental inductance}$$

$$L_E = \frac{2W}{i^2} : \text{energy-equivalent inductance}$$

$W$  being the energy stored in the magnet

$$L_d = L_a + i \frac{dL_a}{di}$$

Differential inductance as a function of current and apparent inductance

- only in ideal case inductance is a constant and the three definitions coincide

# Concept of Inductance: frequency domain

- Inductance will be now assumed constant with respect to current
- Only “small variations” in frequency domain will be considered
- How current perturbations (that can be somehow controlled) affect field variations (in turn affecting the beam) ?
- Let us consider the relative variations of  $\phi$  ,  $v$  and  $i$  :

$$\frac{\Delta\phi}{\phi} = \frac{\phi(f)}{\phi_{nom}} : \text{relative variation of "magnet" magnetic flux}$$

$$\frac{\Delta i}{i} = \frac{i(f)}{i_{nom}} : \text{relative variation of converter current}$$

$$\frac{\Delta v}{v} = \frac{v(f)}{v_{nom}} : \text{relative variation of converter voltage}$$

# Concept of Inductance: frequency domain

□ Let us consider now different sensitivity functions:

$$\frac{\delta\phi}{\delta i} = \frac{\frac{\Delta\phi}{\phi}}{\frac{\Delta i}{i}} : \text{Sensitivity of (relative) flux variation to (relative) current variation}$$

$$\frac{\delta i}{\delta v} = \frac{\frac{\Delta i}{i}}{\frac{\Delta v}{v}} : \text{Sensitivity of (relative) current variation to (relative) voltage variation}$$

$$\frac{\delta\phi}{\delta v} = \frac{\frac{\Delta\phi}{\phi}}{\frac{\Delta v}{v}} : \text{Sensitivity of (relative) flux variation to (relative) voltage variation}$$

□ Or analogously in terms of ppm:

$$\frac{\delta\phi}{\delta i} : \text{ppm of flux variation per ppm of current variation}$$

$$\frac{\delta i}{\delta v} : \text{ppm of current variation per ppm of voltage variation}$$

$$\frac{\delta\phi}{\delta v} : \text{ppm of flux variation per ppm of voltage variation}$$

# Concept of Inductance: frequency domain

□ Let us assume now the following identity:

$$\phi(f) = L(f)i(f)$$

□ it is not exactly what physically happens, but rather what is “observed” at the circuit terminals

$$\phi_{nom} = L(0)I_{nom} = L_{DC}I_{nom}$$

□ In terms of flux then:

$$\frac{\Delta\phi}{\phi} = \frac{L(f)i(f)}{L_{DC}I_{nom}} = \frac{L(f)}{L_{DC}} \frac{i(f)}{I_{nom}} = \frac{L(f)}{L_{DC}} \frac{\Delta i}{i}$$

$$\frac{\delta\phi}{\delta i} = \frac{\frac{\Delta\phi}{\phi}}{\frac{\Delta i}{i}} = \frac{L(f)}{L_{DC}}$$

The sensitivity of flux with respect to current depends on how inductance changes with frequency

# Inductance within a control loop

- It is always assumed that the greater the inductance the better in terms of noise on the current and hence on the B field
- But is it really so?
  - let us consider the open loop case first:

$$v_{ol}(f) = [R + j2\pi fL(f)]i_{ol}(f)$$

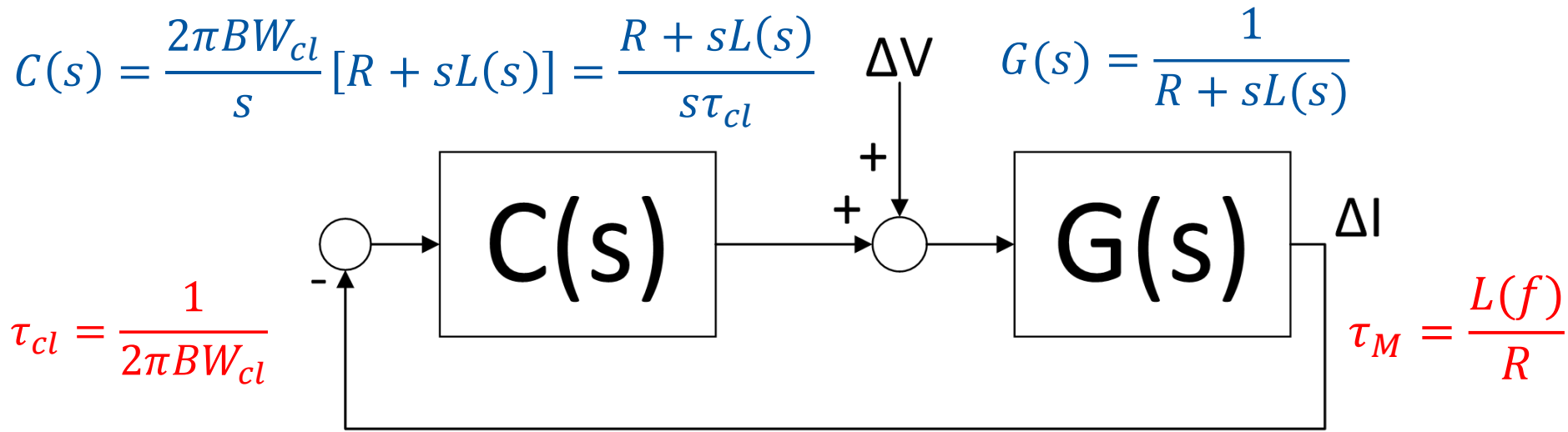
$$\left(\frac{\delta i}{\delta v}\right)_{ol} = \frac{1}{1 + j2\pi f \frac{L(f)}{R}} = \left\{ \left(\frac{\Delta i}{i}\right)_{ol} / \left(\frac{\Delta v}{v}\right)_{ol} = \frac{i_{ol}(f)}{I_{nom}} \frac{RI_{nom}}{[R + j2\pi fL(f)]i_{ol}(f)} \right\}$$

$$\left(\frac{\delta \phi}{\delta v}\right)_{ol} = \frac{\delta \phi}{\delta i} \left(\frac{\delta i}{\delta v}\right)_{ol} = \frac{L(f)}{L_{DC}} \frac{1}{1 + j2\pi f \frac{L(f)}{R}}$$

- the greater  $L(f)$  the greater the “equivalent” time constant, so the greater  $L$  the better in terms of impact of voltage ripple

# Inductance in closed loop

- What happens in closed loop?
  - It depends **strongly** on the **controller!**
  - However some general considerations can be highlighted by means of an ideal “**PI like**” controller



$$\left(\frac{\Delta I}{\Delta V}\right)_{cl} = \frac{1}{R} \frac{j2\pi f \tau_{cl}}{(1 + j2\pi f \tau_M)(1 + j2\pi f \tau_{cl})} = \left\{ \frac{\frac{1}{R + j2\pi f L(f)}}{1 + \frac{C(j2\pi f)}{R + j2\pi f L(f)}} \right\}$$



# Inductance in closed loop

- Now **sensitivity of current to voltage disturbances** is a band-pass:
  - with **mid-frequency gain**  $\frac{\tau_{cl}}{\tau_M} = \frac{BW_{ol}}{BW_{cl}}$
  - normalized by the circuit resistance (**ppm of voltage to ppm of current**)

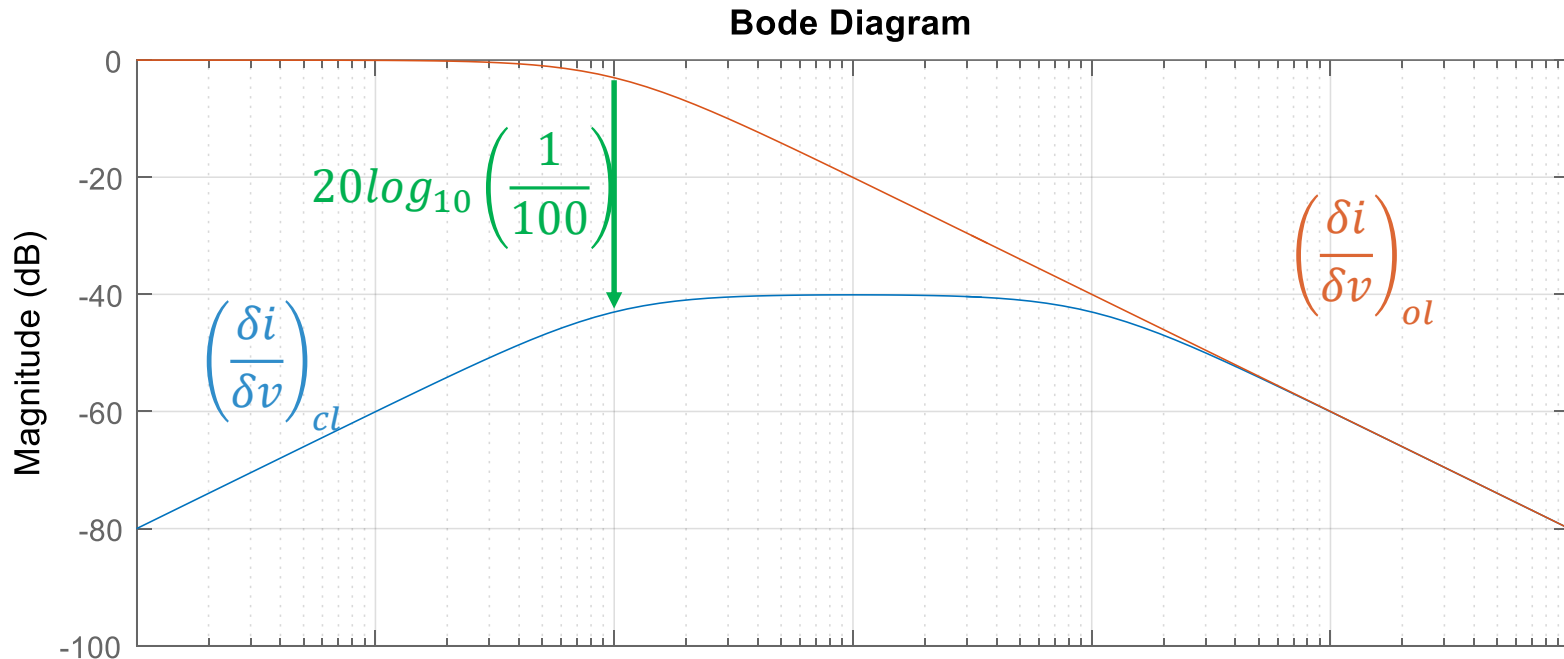
$$\left(\frac{\delta i}{\delta v}\right)_{cl} = \frac{1}{2\pi BW_{cl}} \frac{j2\pi f}{(1 + j2\pi f \tau_M)(1 + j2\pi f \tau_{cl})}$$

- Analogously for the **voltage disturbances to flux**:

$$\left(\frac{\delta \phi}{\delta v}\right)_{cl} = \frac{\delta \phi}{\delta i} \left(\frac{\delta i}{\delta v}\right)_{cl} = \frac{L(f)}{L_{DC}} \frac{j2\pi f \tau_{cl}}{(1 + j2\pi f \tau_M)(1 + j2\pi f \tau_{cl})}$$

# Inductance in closed loop

□ Assuming a **reasonable** ratio  $\frac{\tau_{cl}}{\tau_M^{DC}} = \frac{BW_{ol}}{BW_{cl}} = \frac{1}{100}$  and  $L(f) = L_{DC}$  :



In low and mid frequency ranges the “original” magnet inductance is irrelevant, only the control-loop parameters matter.

For  $f > BW_{cl}$  however the attenuation is given by the “original” magnet inductance, so it would seem that the higher the inductance the better!

But in high frequency the “inductance” might drop with frequency!

# How inductance variation with frequency is observed

- Assuming ideal **RL-series** model for the magnet's impedance:

$$Z(f) = R + j2\pi fL$$

- $L$  can be estimated from measured  $Z$  as:  $\hat{L} = \frac{\text{imag}(Z)}{2\pi f} = \frac{Z-R}{j2\pi f}$

- $Z - R$  is never purely imaginary for real inductors (\*)

- it is not correct to consider a “frequency varying inductance”, but:

- $|\hat{L}|$  decreases with frequency

- it is therefore reasonable to deduce  $\frac{L(f)}{L_{DC}} \leq 1$ , hence:  $\frac{\delta\phi}{\delta i} \leq 1$

- **the ppm of flux are always less than the ppm of current !**

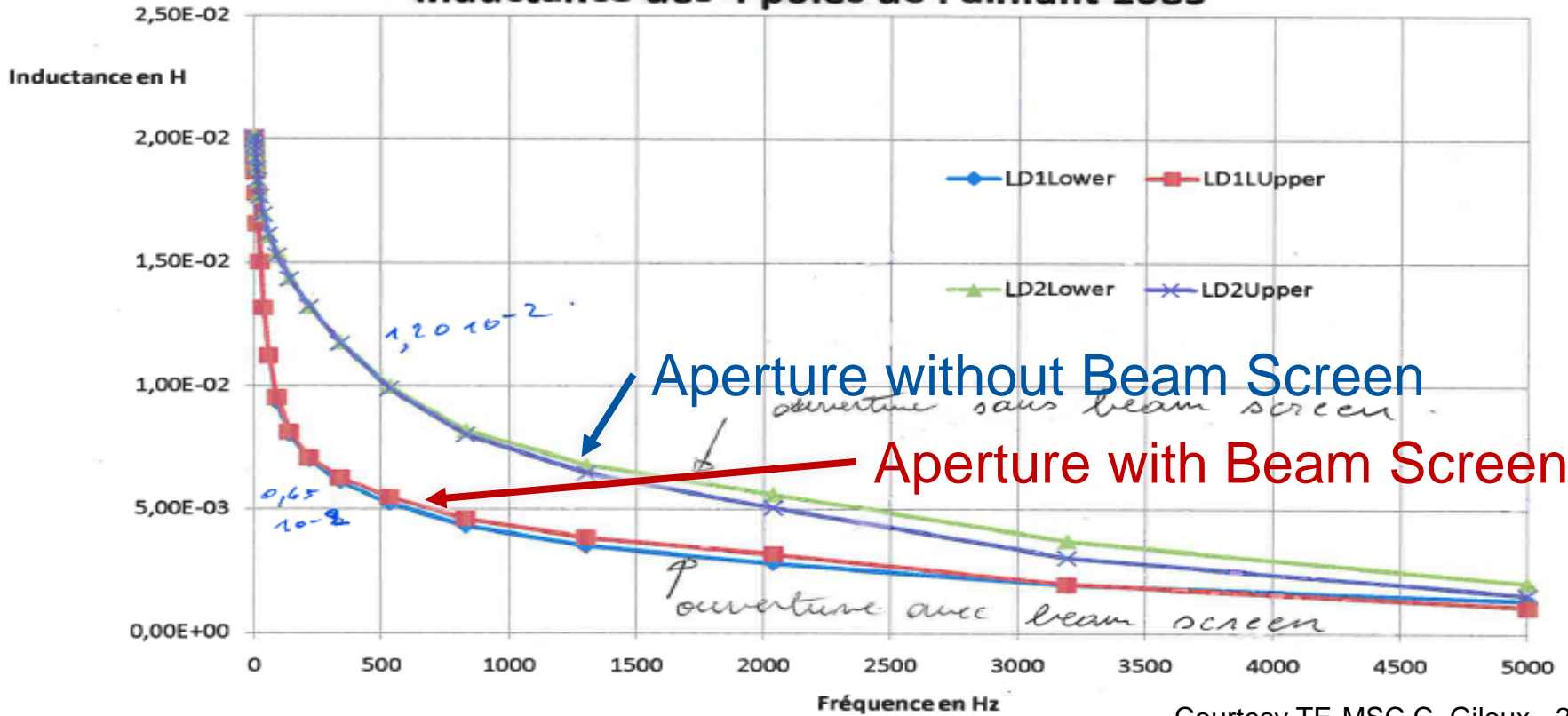
(\*) Kramers-Kronig relationships between real and imaginary parts due to causality

# So what is really going on?

- ❑ The PC powers not only the magnet but also losses:
  - ❑ well known eddy currents
  - ❑ they affect differently resistive and superconducting magnets
  - ❑ focus here will be only on superconducting magnets
    - ❑ magnets themselves have losses
    - ❑ cold beam screens play a dominant role
- ❑ only brief references for normal conducting (lack of time...)

# A SC example: LHC Magnet at 1.9 K

## Inductance des 4 pôles de l'aimant 1085



Courtesy TE-MS C. Giloux - 2009

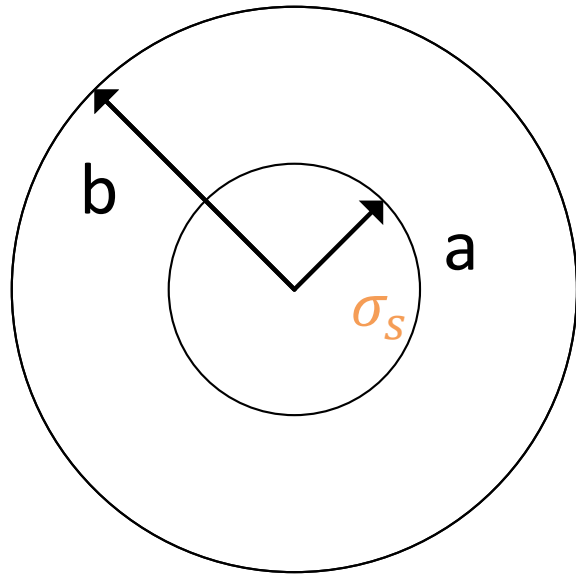
No DC bias

Small signal excitation

# Ideal $n$ -pole with inner conductive shell (infinitely thin)

$$A_z(r, \theta, f) = \mu_0 \frac{b}{2n} \left(\frac{r}{b}\right)^n J_z(b, \theta, f)$$

$$J_z(b, \theta, f) = \frac{N_c I_0}{4b} \cos(n\theta) \sin(2\pi f t)$$



Induced currents appear in the shell:

$$J'_z(a, \theta, f) = \sigma_s E_z = -j2\pi f \sigma_s [A_z(a, \theta, f) + A'_z(a, \theta, f)]$$

Contributing to the total vector potential as:

$$A_z^{tot}(r, \theta, f) = A_z(r, \theta, f) + A'_z(r, \theta, f) = \frac{A_z(r, \theta, f)}{1 + j2\pi f \tau_n}$$

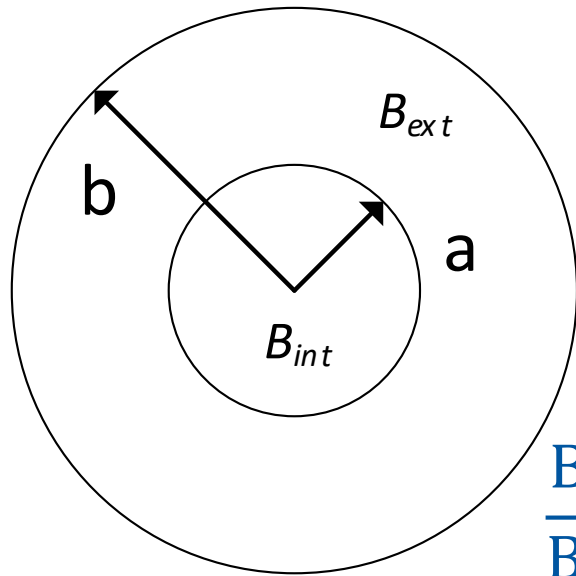
$$\tau_n = \sigma_s \mu_0 \frac{a}{2n}$$

$$f_n = \frac{1}{2\pi \tau_n} = \frac{2n}{\mu_0 2\pi a \sigma_s}$$

**Cut-off frequency increases with the order of the magnet:** so for a given beam screen the shielding for a dipole is twice more effective than for a quadrupole and so on...

Notation and calculations taken directly from [2] – rewritten to highlight where the pole comes from

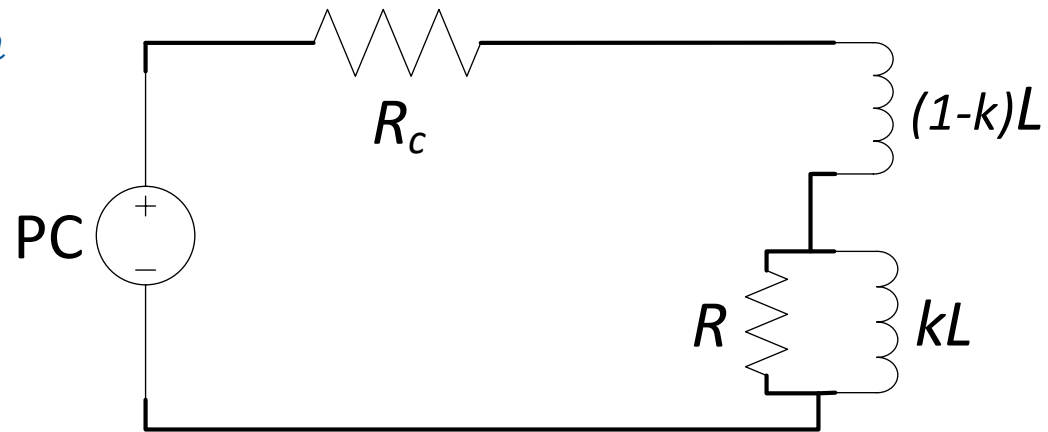
# Ideal $n$ -pole with inner conductive shell (infinitely thin)



$$k = \left(\frac{a}{b}\right)^{2n}$$

$$R = \frac{kL}{\tau_n}$$

$$\frac{B_{int}}{B_{ext}} = \frac{1}{1 + s\tau_n}$$

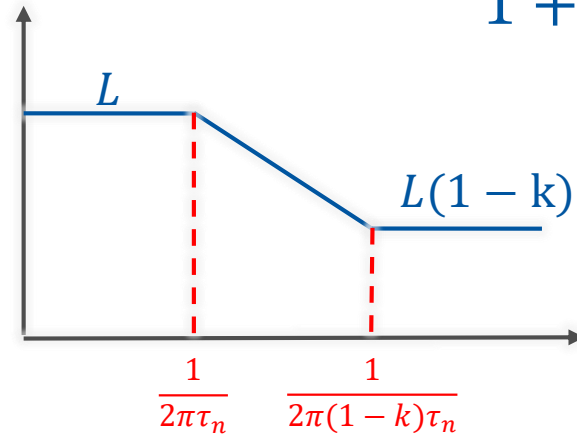


$$L_{eq}(s) = L \frac{1 + s\tau_n(1 - k)}{1 + s\tau_n}$$

$$L_{eq}(0) = L$$

$$L_{eq}(\infty) = L(1 - k)$$

$$Z(s) = R_c + sL \frac{1 + s\tau_n(1 - k)}{1 + s\tau_n}$$

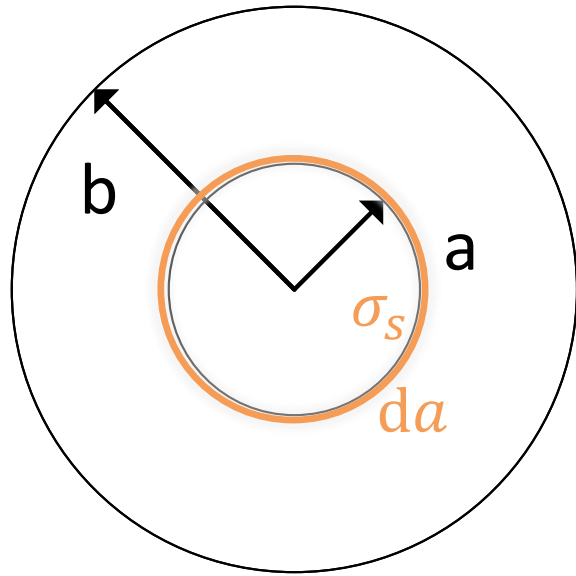


Notation and calculations taken directly from [2] – rewritten to highlight the “equivalent inductance”

# Ideal $n$ -pole with **thin** inner conductive shell

$$A_z(r, \theta, f) = \mu_0 \frac{b}{2n} \left(\frac{r}{b}\right)^n J_z(b, \theta, f)$$

$$J_z(b, \theta, f) = \frac{N_c I_0}{4b} \cos(n\theta) \sin(2\pi f t)$$



$$J'_z(a, \theta, f) = \sigma_s E_z = -j2\pi f \sigma_s [A_z(a, \theta, f) + A'_z(a, \theta, f)]$$

$$A_z^{tot}(r, \theta, f) = A_z(r, \theta, f) + A'_z(r, \theta, f) \cong \frac{A_z(r, \theta, f)}{1 + j2\pi f \tau_n}$$

$$\tau_n = \sigma_s \mu_0 \frac{a}{2n} \quad f_n = \frac{1}{2\pi \tau_n} = \frac{2n}{\mu_0 2\pi a \sigma_s} \quad \sigma_s = \frac{\sigma da}{A} \quad f_n = \frac{2n}{\mu_0 \underbrace{2\pi a da}_{A} \sigma} = \frac{2n}{\mu_0 A \sigma}$$

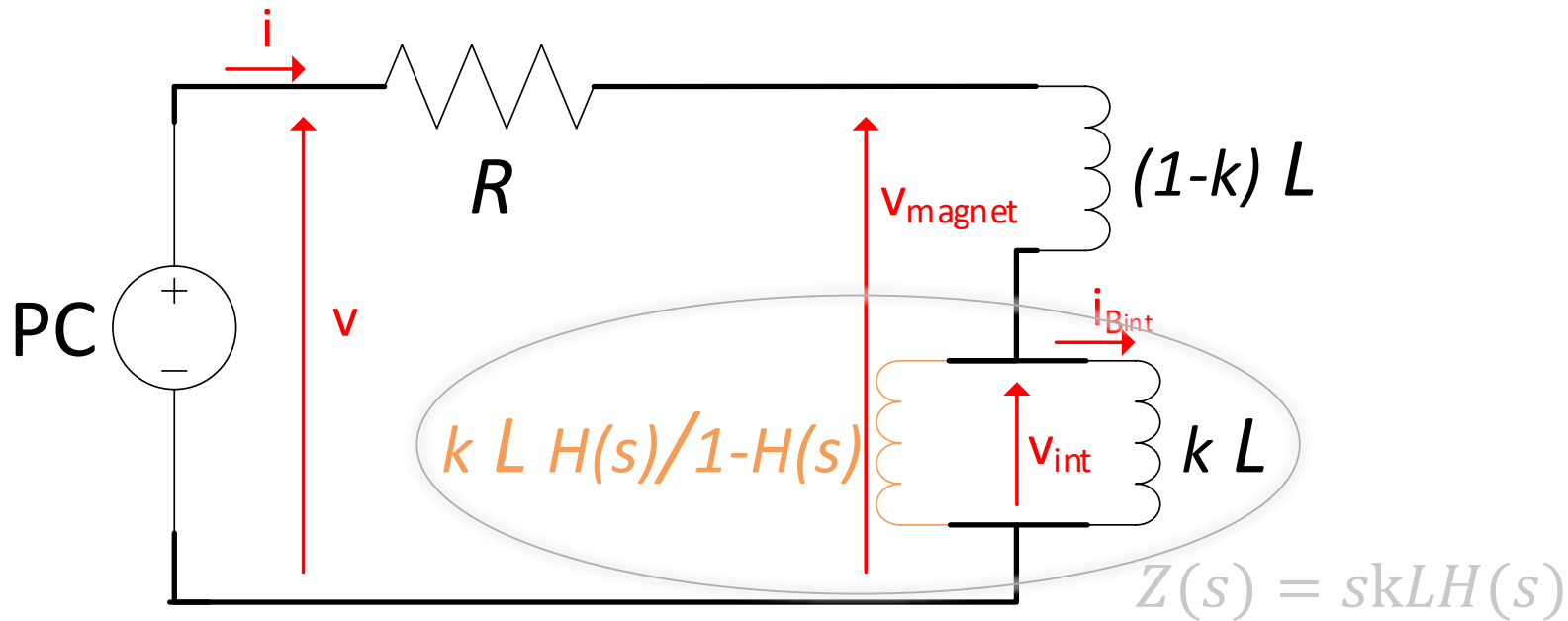
At first approximation the **cut-off** is **directly proportional** to  $n$  and **inversely proportional** to the product  $A\sigma$  of the **cross-section area** of the beam screen and its **conductivity**.

For thick and/or non-cylindrical BS the magnetic TF  $H(s)$  is not a simple pole!



# Full admittance model with an arbitrary beam screen

$$\frac{i}{v} = \frac{1}{R + sL [(1-k) + kH(s)]} = \frac{1}{R} \frac{1}{1 + s\tau [(1-k) + kH(s)]}$$



It is also possible to estimate the “**trans-admittance**” between the **useful current** (which is actually **producing** the **B field** seen by the beam) and the PC voltage!

$$\frac{i_{B_{int}}}{v} = \frac{i_{B_{int}}}{v_{int}} \frac{v_{int}}{v_{magnet}} \frac{v_{magnet}}{v}$$

$$\frac{i_{B_{int}}}{v} = \frac{1}{s k L} \frac{s k L H(s)}{s L [(1-k) + k H(s)]} \frac{s L [(1-k) + k H(s)]}{R + s L [(1-k) + k H(s)]} = \frac{1}{R} \frac{H(s)}{1 + s\tau [(1-k) + k H(s)]}$$

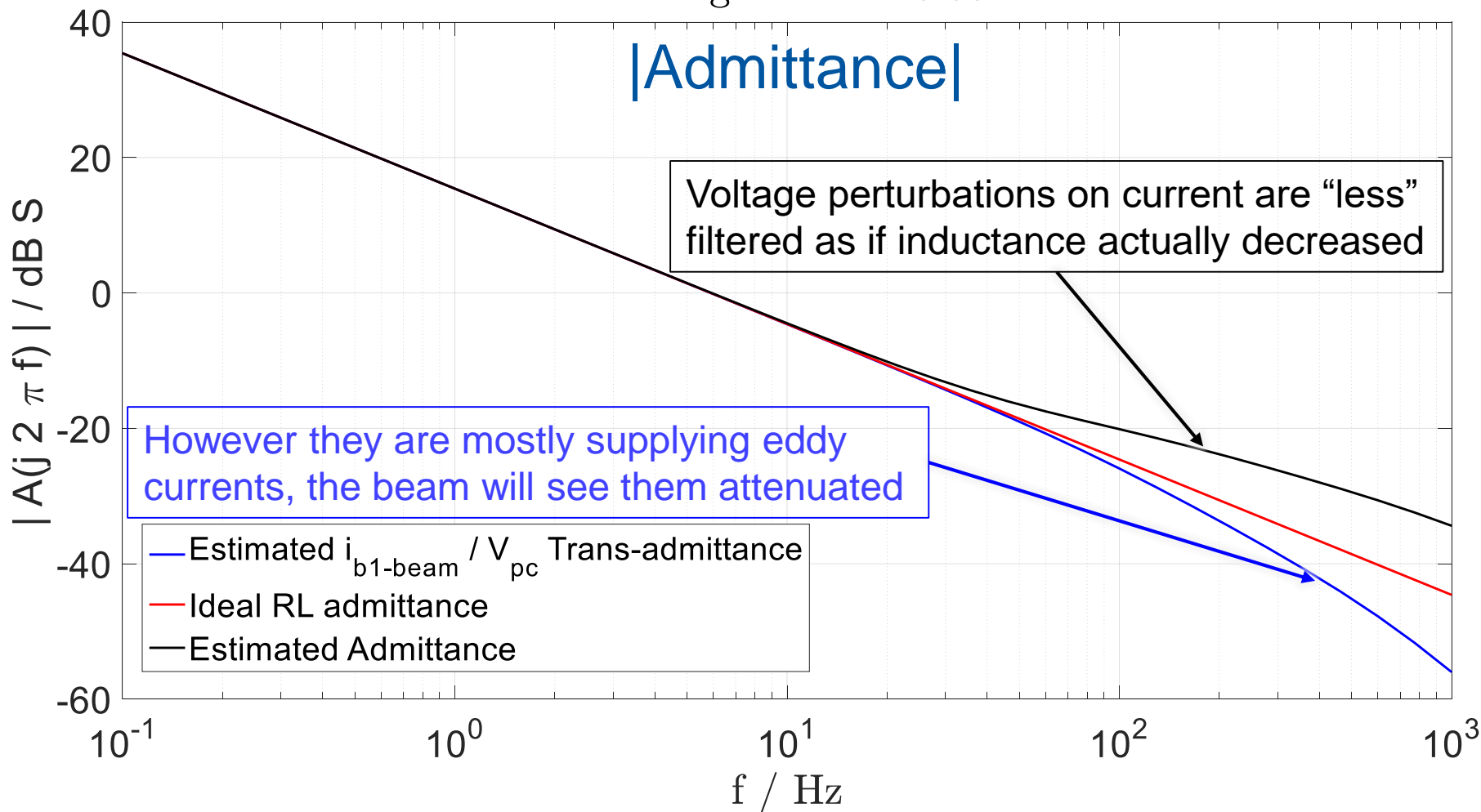
# Ideal Dipole + Beam Screen model – D1 case

$$R_c = 0.27 \text{ m}\Omega$$

$$L_{DC} = 27 \text{ mH}$$

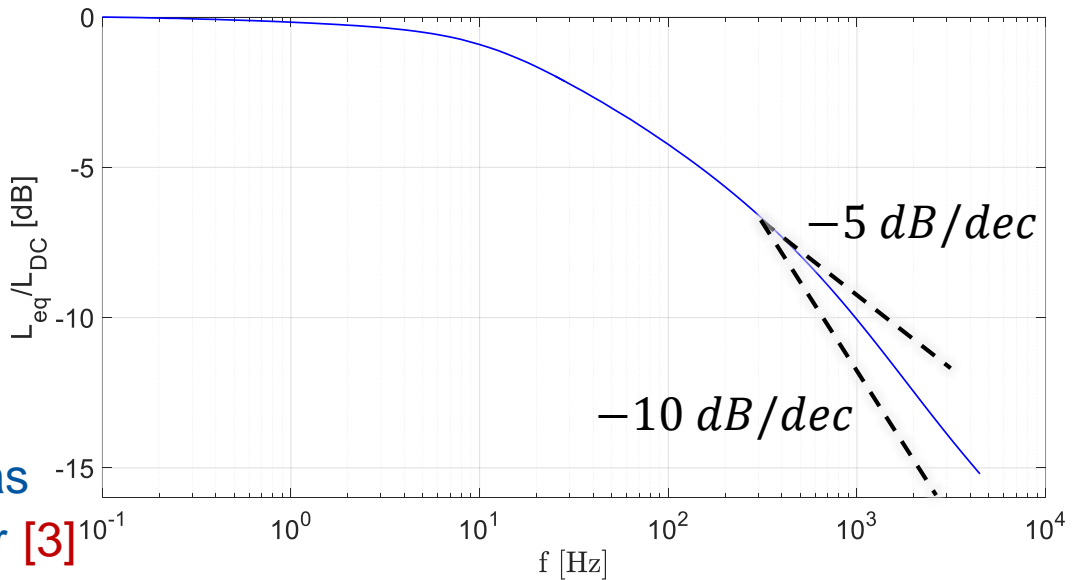
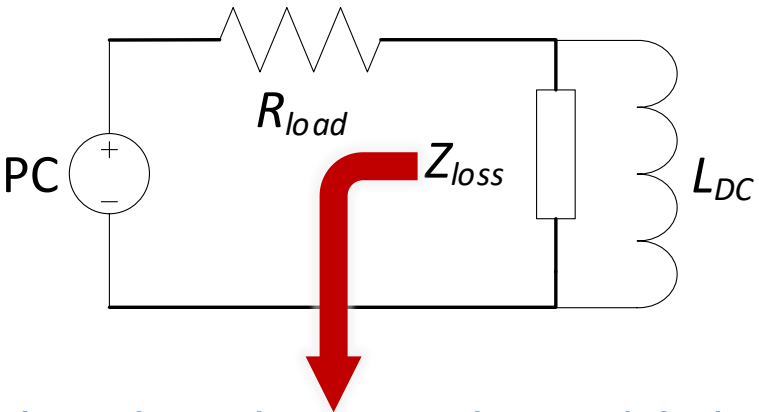
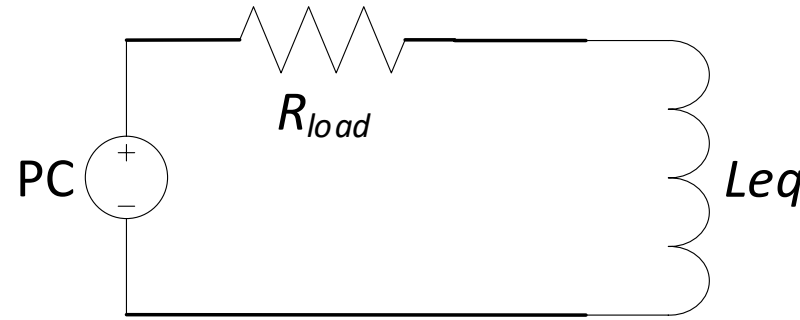
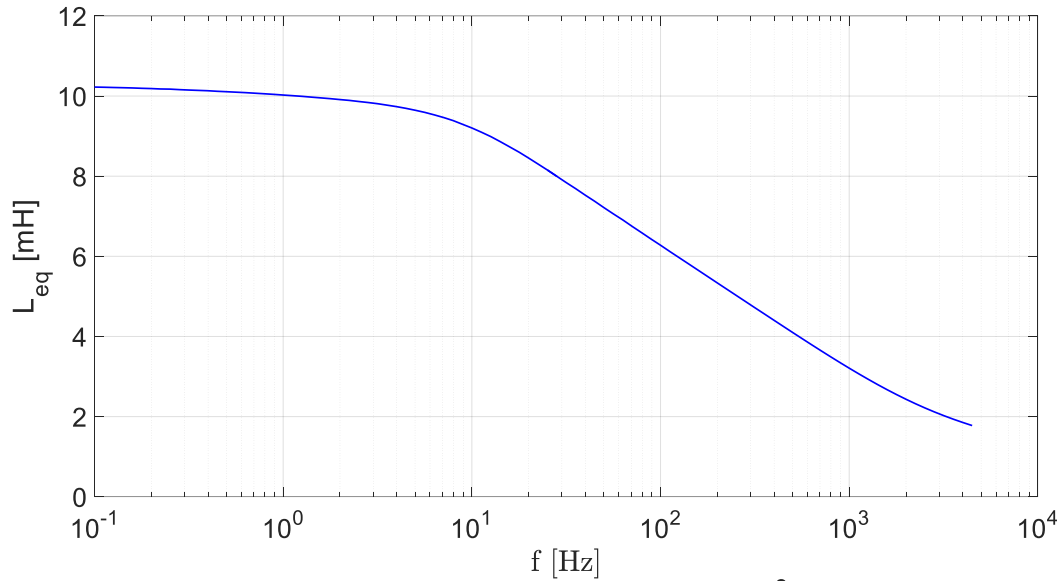
$$\frac{B_{int}}{B_{ext}} = H(s) \cong \frac{1}{1 + s \cdot 2.58 \text{ ms}}$$

D1 magnet -  $k = 0.68$



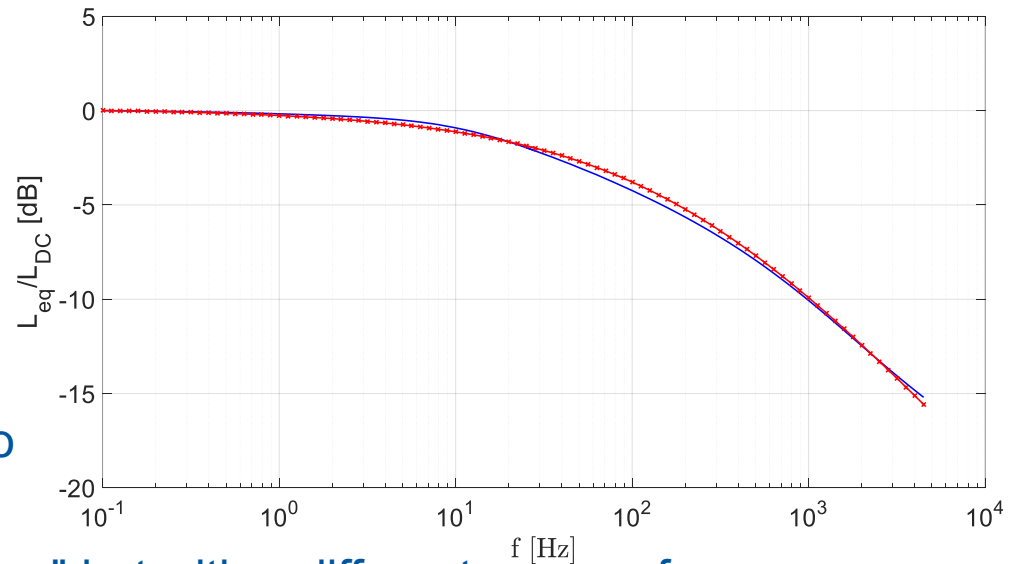
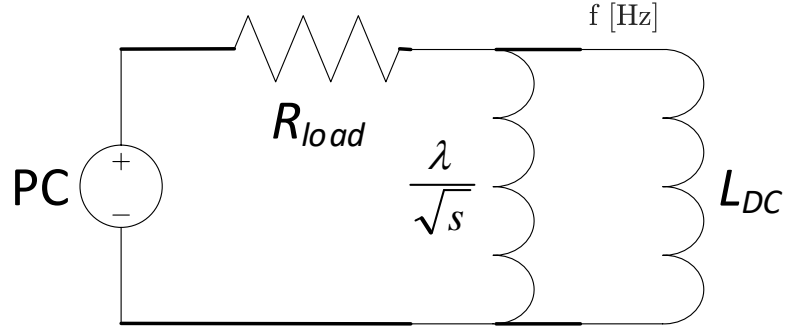
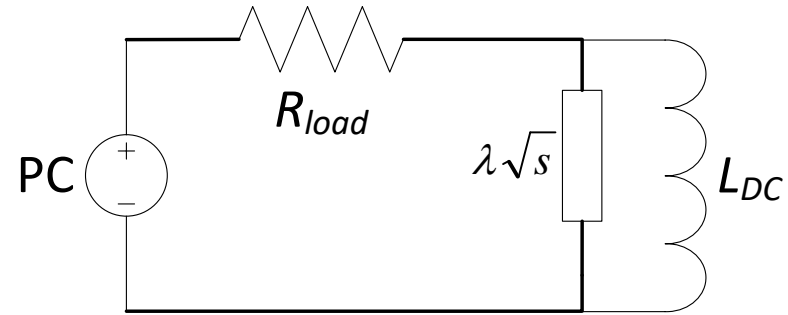
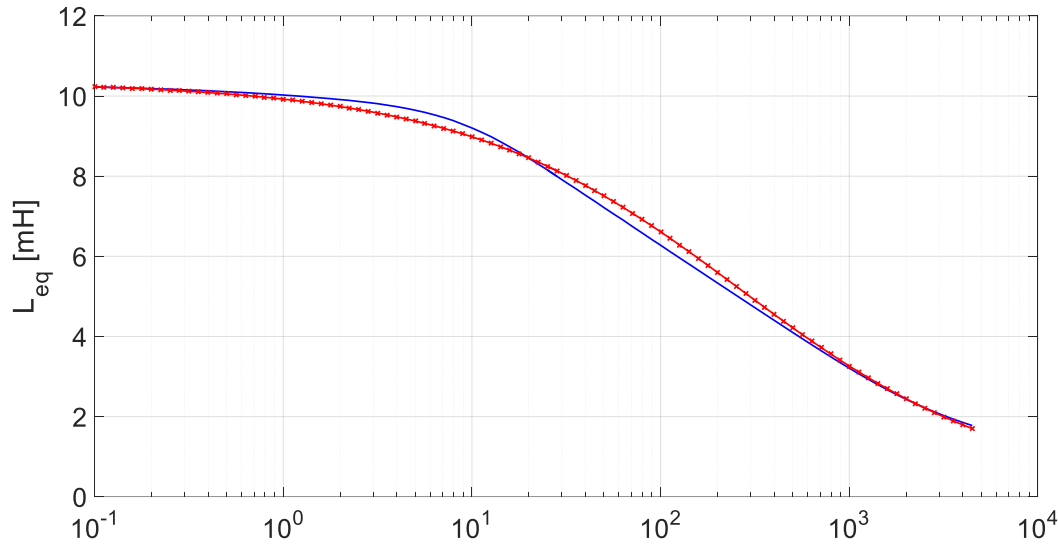
SC magnets also have losses!

# $L_{eq}$ of MQXF magnet (short model) - $Nb_3Sn$



Loss impedance can be modeled as having fractional-order  $s^\alpha$  behavior [3]

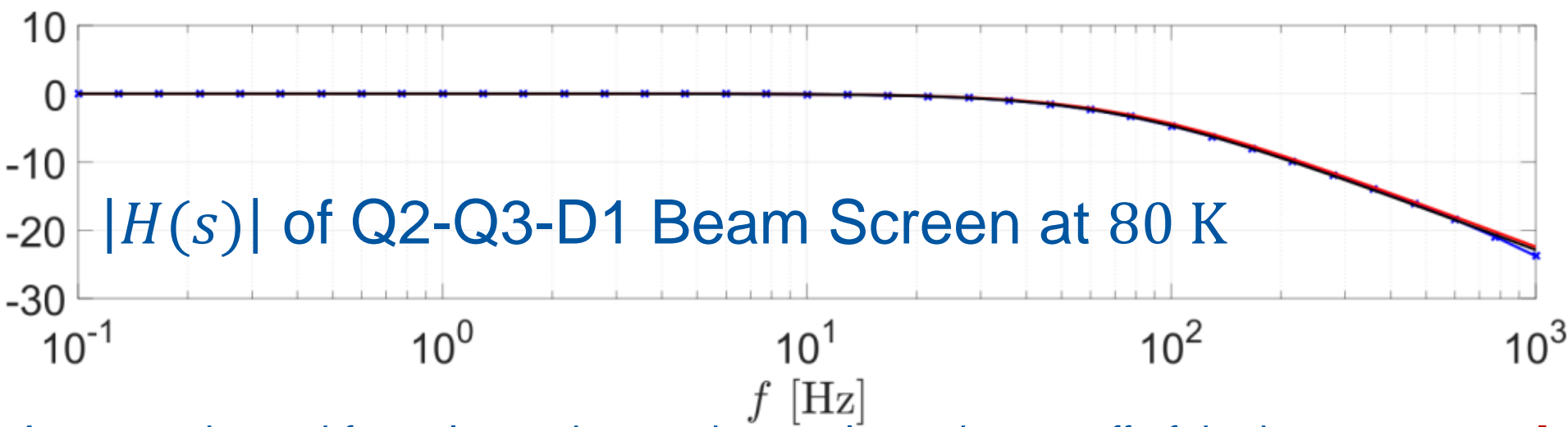
# $L_{eq}$ of MQXF magnet (short model) - $Nb_3Sn$



Assuming impedance goes with  $\sqrt{s}$  a single parameter needed to be fitted to get excellent agreement 😊

Fractional order confirmed also “at warm” but with a different power of  $s$  (not discussed here)

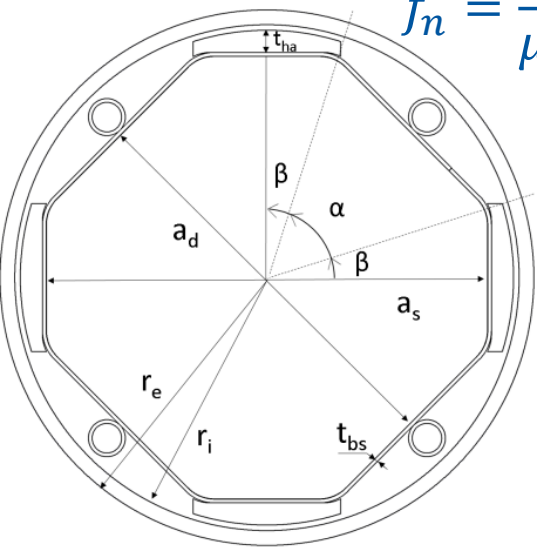
# Cold Beam Screens still play a dominant role!



An approximated formula can be used to evaluate the cut-off of the beam screen [4]

$$f_n = \frac{2n}{\mu_0 A \sigma}$$

For this beam screen (as Q3 - MQXF)  $f_2 = 75.8$  Hz



$$\widehat{A\sigma}_{BS} = 4 \frac{\pi}{2} \left[ \left( c_{\beta}^{(n)} + c_{\bar{\beta}}^{(n)} \right) F_{2\beta} \bar{\rho}_{\beta} (\Delta\sigma)_{\beta} + c_{\alpha}^{(n)} F_{\alpha} \bar{\rho}_{\alpha} (\Delta\sigma)_{\alpha} \right]$$

Magnet losses kick in first but then the losses of the beam screen increase more rapidly! Both elements together “shunt” the high frequency components of the current preventing them to affect the beam!

$$c_{\beta}^{(n)} = \int_0^{\beta} |\cos(n\theta)| d\theta$$

$$c_{\alpha}^{(n)} = \int_{\beta}^{\beta+\alpha} |\cos(n\theta)| d\theta$$

$$c_{\bar{\beta}}^{(n)} = \int_{\beta+\alpha}^{\frac{\pi}{2}-\beta} |\cos(n\theta)| d\theta$$

# Summary

## □ Concept of Inductance

- impact of PC current and voltage on the (magnetic field seen by the) beam

## □ Observed frequency dependence of inductance

- the role of the inductance within a control loop

## □ Losses

- in superconducting magnets themselves
  - loss impedance can be modeled as fractional-order  $s^\alpha$
- the dominant role of cold beam screen in SC magnets
- fractional-order model suitable also for NC magnets

# References

- [1] M. Buzio et al. : Measurement of the inductance of resistive magnets: two case studies – CERN-ATS-Note-2011-047 TECH
- [2] Robert E. Shafer: Eddy Currents, Dispersion Relations, and Transient Effects in Superconducting Magnets, TM-991, Fermilab 1980
- [3] Joseph J. Feeley : A simple Dynamic Model for Eddy Currents in a Magnetic Actuator – IEEE Transactions on Magnetics 1996
- [4] M. Morrone, M. Martino et al : Magnetic Frequency Response of High-Luminosity Large Hadron Collider Beam Screens, submitted to PRAB, 2017
- [4] Gonzales and Brambilla : Frequency dependence of the resistance and inductance of solid core magnets – IEEE Transactions on Nuclear Science 1965





[www.cern.ch](http://www.cern.ch)

# Additional slides

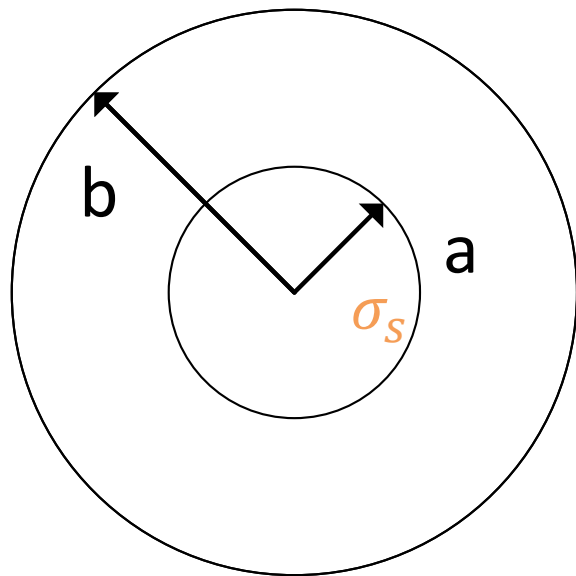


# Beam Screen: Mathematical Derivation



# Ideal $n$ -pole with inner conductive shell (infinitely thin)

$$A_z(r, \theta, f) = \mu_0 \frac{b}{2n} \left(\frac{r}{b}\right)^n J_z(b, \theta, f) \quad J_z(b, \theta, f) = \frac{N_c I_0}{4b} \cos(n\theta) \sin(2\pi f t)$$



$$J'_z(a, \theta, f) = \sigma_s E_z = -j2\pi f \sigma_s [A_z(a, \theta, f) + A'_z(a, \theta, f)]$$

$$\begin{aligned} A'_z(r, \theta, f) &= \mu_0 \frac{a}{2n} \left(\frac{r}{a}\right)^n J'_z(a, \theta, f) \\ &= -j2\pi f \sigma_s \mu_0 \frac{a}{2n} \left(\frac{r}{a}\right)^n [A_z(a, \theta, f) + A'_z(a, \theta, f)] \\ &= -j2\pi f \sigma_s \mu_0 \frac{a}{2n} [A_z(r, \theta, f) + A'_z(r, \theta, f)] \end{aligned}$$

$$A'_z(r, \theta, f) = -\frac{j2\pi f \sigma_s \mu_0 \frac{a}{2n}}{1 + j2\pi f \sigma_s \mu_0 \frac{a}{2n}} A_z(r, \theta, f)$$

$$A_z^{tot}(r, \theta, f) = A_z(r, \theta, f) + A'_z(r, \theta, f) = \frac{A_z(r, \theta, f)}{1 + j2\pi f \sigma_s \mu_0 \frac{a}{2n}} = \frac{A_z(r, \theta, f)}{1 + j2\pi f \tau_n}$$

$$\tau_n = \sigma_s \mu_0 \frac{a}{2n} \quad f_n = \frac{1}{2\pi \tau_n} = \frac{2n}{\mu_0 2\pi a \sigma_s}$$

Cut-off frequency increases with the order of the magnet: so for a given beam screen the shielding for a dipole is twice more effective than for a quadrupole and so on...

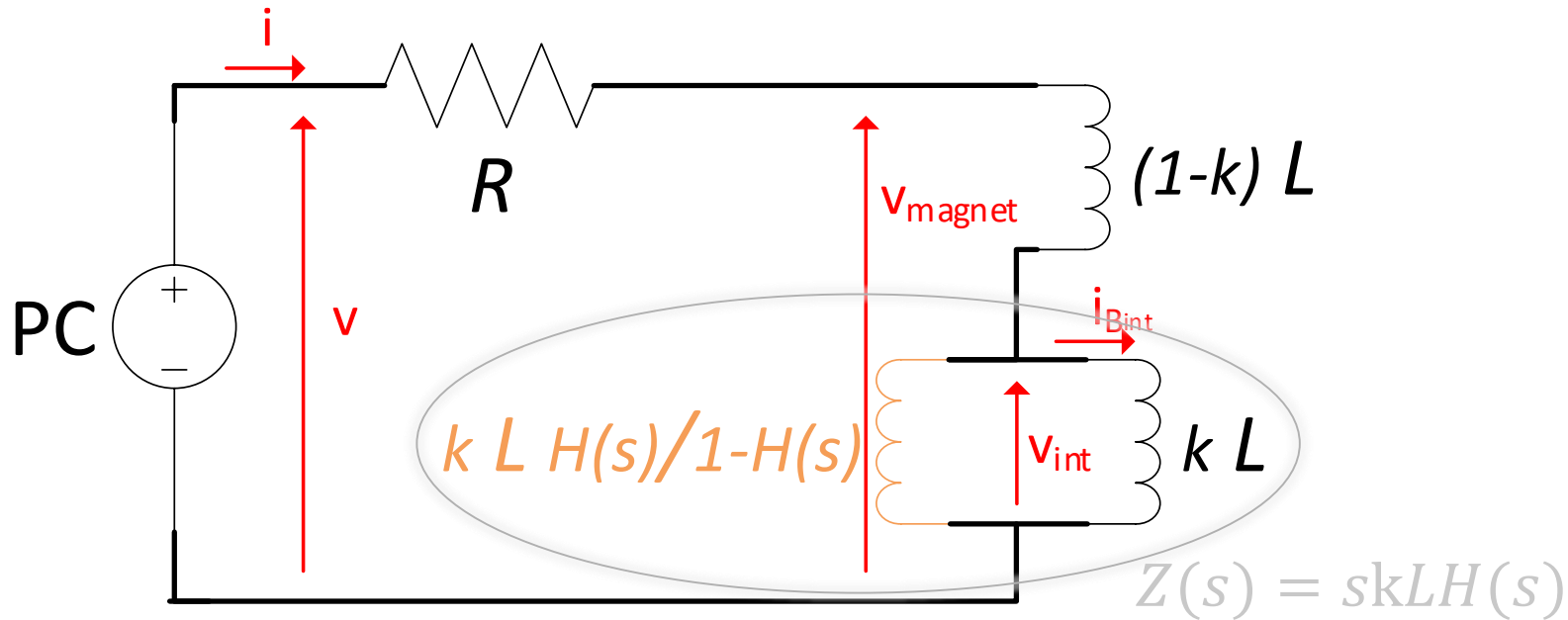
Notation and calculations taken directly from [XX] – rewritten to highlight where the pole comes from

# Beam Screen Equivalent Circuit



# Full admittance model with an arbitrary beam screen

$$\frac{i}{v} = \frac{1}{R + sL[(1-k) + kH(s)]} = \frac{1}{R} \frac{1}{1 + s\tau[(1-k) + kH(s)]}$$



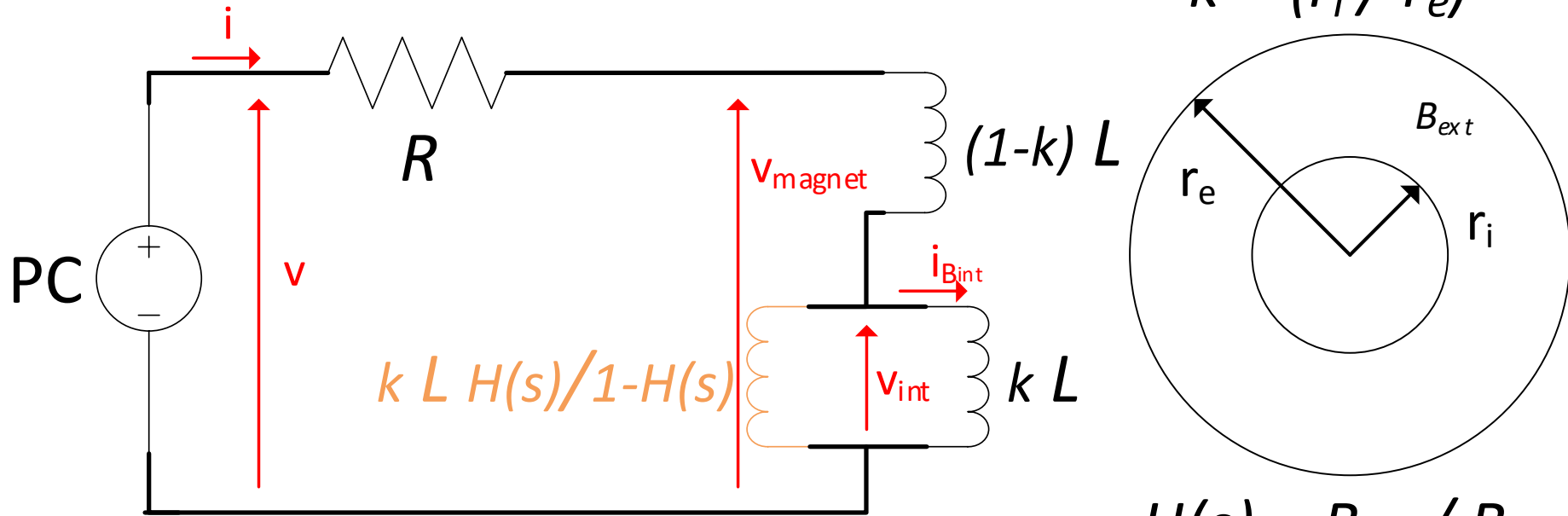
It is also possible to estimate the “trans-admittance” between the **useful current** (which is actually **producing** the **B field** seen by the beam) and the PC voltage!

$$\frac{i_{B_{int}}}{v} = \frac{i_{B_{int}}}{v_{int}} \frac{v_{int}}{v_{magnet}} \frac{v_{magnet}}{v} \quad \frac{i_{B_{int}}}{v_{int}} = \frac{1}{s k L} \quad \frac{v_{int}}{v_{magnet}} = \frac{s k L H(s)}{s L [(1-k) + k H(s)]} \quad \frac{v_{magnet}}{v} = \frac{s L [(1-k) + k H(s)]}{R + s L [(1-k) + k H(s)]}$$

$$\frac{i_{B_{int}}}{v} = \frac{1}{s k L} \frac{s k L H(s)}{s L [(1-k) + k H(s)]} \frac{s L [(1-k) + k H(s)]}{R + s L [(1-k) + k H(s)]} = \frac{1}{R} \frac{H(s)}{1 + s\tau [(1-k) + k H(s)]}$$

# Full admittance model with an arbitrary beam screen

$$k = (r_i / r_e)^{2n}$$



$$H(s) = B_{int} / B_{ext}$$

$Z_{//}(s) = skL \frac{H(s)}{1-H(s)}$  it is actually an impedance as  $real\{Z_{//}(s)\} \geq 0$  for  $real\{s\} \geq 0$

$$skL // Z_{//}(s) = skLH(s)$$

$$\frac{v_{int}}{v_{magnet}} = \frac{skLH(s)}{sL[(1-k) + kH(s)]} = \frac{kH(s)}{[(1-k) + kH(s)]}$$

if  $H(s) = 1$  there is **only a geometrical factor!**

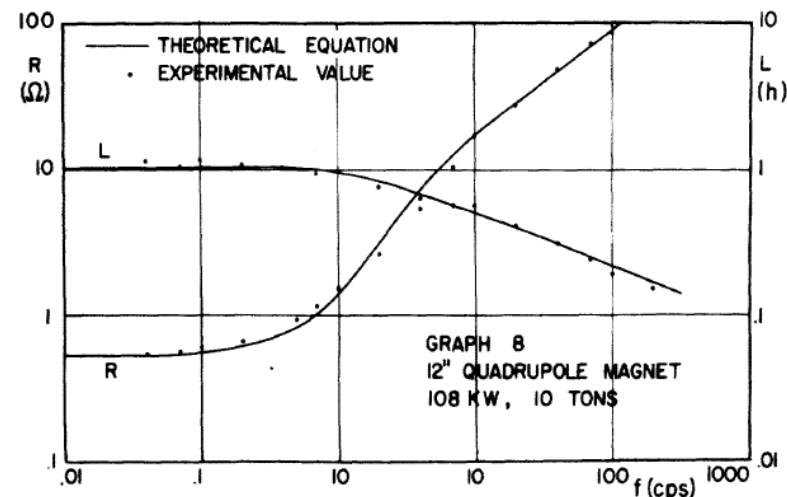
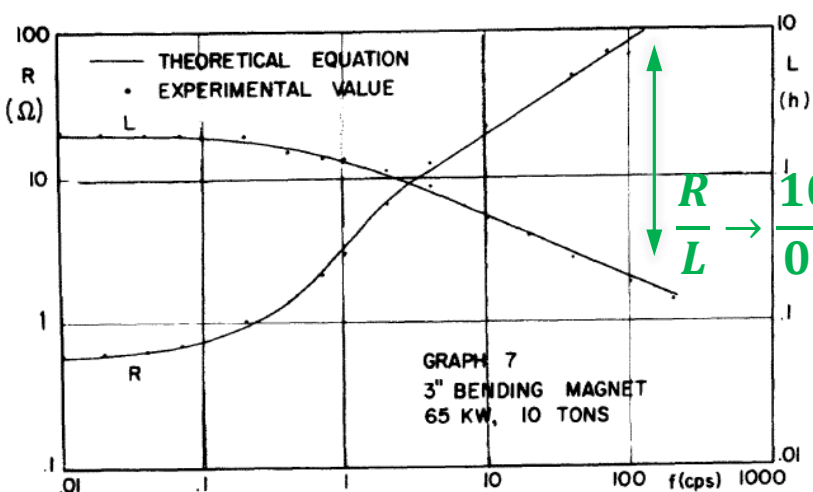
At the end everything goes as if the inductance were  $L[(1-k) + kH(s)]$ :

- in low frequency  $H(s) \cong 1$  so the PC sees the whole inductance  $L$
- in high frequency  $H(s) \rightarrow 0$  so the PC sees only the fraction  $L(1-k)$

# Normal conducting magnets



# Some results for Normal Conducting Magnets



$$|\phi_{circuit}(f)| = \frac{v_{circuit}(f)}{\left| \frac{R}{L_{ac}}(f) + j2\pi f \right|} \leq \frac{v_{circuit}(f)}{2\pi f}$$

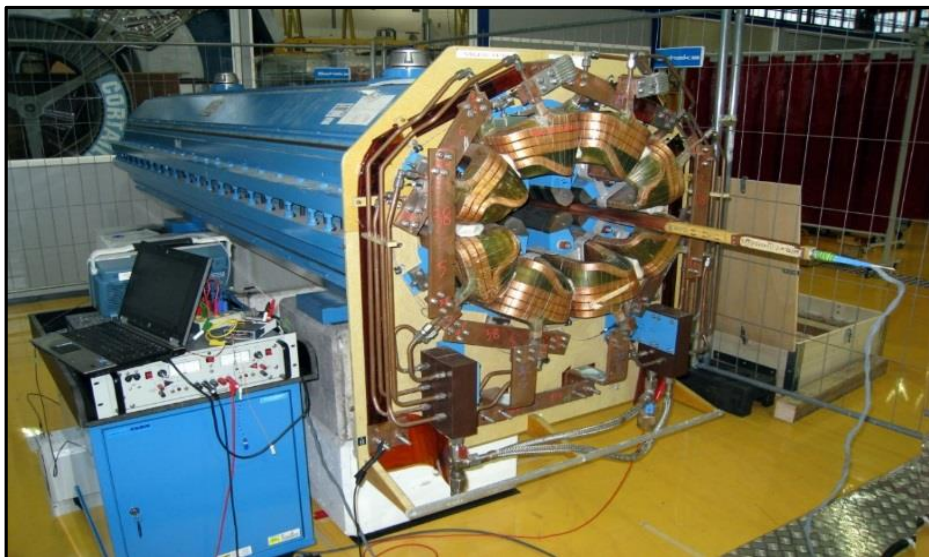
Gonzales and Brambilla: Resistance and Inductance of Solid Core Magnets – IEEE Transactions on Nuclear Science – 1965

# Some results for Normal Conducting Magnets

With vacuum chamber

No DC bias

Very small excitation current



The actual **Integrated Gradient** was measured as a function of the **frequency**

